

Anomalous Field Enhancement in Planar Semiconducting Cold Cathodes from Spontaneous Ordering in the Accumulation Region

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ABSTRACT

Wide band gap semiconductors exhibit a low electron affinity and may prove suitable for cold cathode applications. We introduce a simple closed-form analytic approximation for the stability of electrons in the electron accumulation layer of planar Low Electron Affinity (LEA) semiconducting cathodes. This analysis extends our previous results, which used Runge-Kutta numerical integration of the linearized equations of motion for the electric potential and quasi Fermi level. The model shows conditions in which the electrons in the accumulation layer form a two dimensional array of regions of higher and lower electron density. This instability could lead to field enhancement without surface roughness and could account for observed electron emission at low applied fields.

INTRODUCTION

Wide band gap semiconductors such as diamond, AlN, or BN can exhibit negative electron affinities depending on surface termination. However, n-type doping of these materials is difficult. In contrast, $\text{Al}_{1-x}\text{Ga}_x\text{N}$ alloys ($x \sim 0.5$) can be doped n-type and can exhibit a low electron affinity of ~ 1 eV [1]. LEA-coated cold cathodes may lead to vacuum electron devices (VEDs) with unprecedented versatility and performance [2]. Robust electron emission with high current density has been predicted for graded aluminum gallium nitride that is undoped [3] or doped [4]. LEA cathodes that require neither field enhancement nor high temperature may lead to VEDs with micrometer grid-to-cathode distances, picosecond transit times, and terahertz operating frequencies.

In this paper we provide simple closed-form expressions for our previous numerical prediction that LEA cathodes may be unstable against certain three-dimensional perturbations of the electron accumulation layer [5]. The instabilities may lead to ordered regions of higher or lower electron densities, and these regions would result in field enhancement without surface morphology.

Our effort is motivated by recent characterizations of planar cathodes which emit more electrons at lower voltages than can be readily explained without field enhancement [6] according to the usual equilibrium theory [7].

MODEL

In a Cartesian coordinate system, consider a uniform n-type semiconductor filling the region $x \leq 0$ with vacuum in the region $x > 0$ as shown in Figure 1.

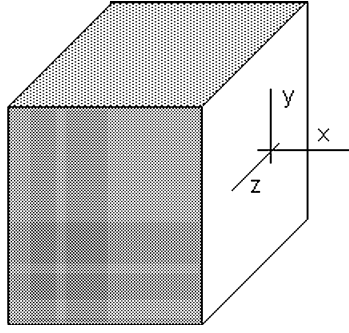


Figure 1. Coordinate system at the semiconductor/vacuum interface of a semiconductor filling the half space $x < 0$. The anode (not shown) is parallel to the interface at some $x > 0$ in the vacuum.

Assuming complete ionization and Maxwell-Boltzmann statistics, the electron density for $x \leq 0$ is

$$n = N_C \exp\{ (E_{Fn} - E_C) / k_B T \}$$

where N_C is the effective density of states of the conduction band and E_{Fn} is the quasi Fermi level for electrons. E_{Fn} differs from the bulk equilibrium Fermi level E_F by

$$\phi = (E_{Fn} - E_F) / q \quad (1)$$

measured in volts. The minimum of the conduction band E_C can be defined to be zero in the bulk, so that

$$\psi = - E_C / q$$

is the usual electric potential in volts. Then

$$n = N_D \exp\{ (\psi + \phi) / V_T \}$$

where $V_T = k_B T / q$ is the usual thermal voltage. The deviations ψ and ϕ obey Poisson's equation $\nabla^2 \psi = (q / \epsilon_S) (N_D - n)$ which is now

$$\nabla^2 \psi = (q N_D / \epsilon_S) [\exp\{(\psi + \phi) / V_T\} - 1]$$

in standard units where ϵ_S is the dielectric permittivity of the semiconductor and $q > 0$ is the fundamental charge. The deviations also conserve charge

$$-q \partial n / \partial t + \nabla \cdot \mathbf{J} = 0$$

where the vector current density due to drift and diffusion is $\mathbf{J} = q \mu n \mathbf{F} + q D \nabla n$, which simplifies to

$$\mathbf{J} = q \mu n \nabla \phi,$$

if the electron mobility μ and the diffusion coefficient $D = \mu V_T$ are treated as independent of electric field

$$\mathbf{F} = -\nabla\psi.$$

The divergence of \mathbf{J} can then be obtained from the product rule

$$\nabla \cdot \mathbf{J} = q \mu (n \nabla^2 \phi + \nabla n \cdot \nabla \phi)$$

and, since $\partial n / \partial t = (n/V_T) \partial / \partial t (\psi + \phi)$, the continuity equation can be written as

$$\mu V_T (\nabla \cdot N_D \exp\{ (\psi + \phi) / V_T \} \nabla \phi) = \partial / \partial t N_D \exp\{ (\psi + \phi) / V_T \}$$

which can be simplified by performing $\partial / \partial t$ and factoring out the exponential to obtain

$$\mu V_T \{ \nabla^2 \phi + \nabla(\psi + \phi) \cdot \nabla \phi / V_T \} = \partial / \partial t (\psi + \phi)$$

in standard units. We will measure lengths in units of the extrinsic Debye length

$$L_D = [\epsilon_S V_T / q N_D]^{1/2},$$

measure times in units of the dielectric relaxation time for electrons in the bulk

$$\tau_R = \epsilon_S / q \mu N_D,$$

and measure voltages in units of V_T , so that Poisson's equation becomes

$$\nabla^2 \psi = \exp(\psi + \phi) - 1 \quad (2)$$

and the continuity equation becomes

$$\nabla^2 \phi + \nabla(\psi + \phi) \cdot \nabla \phi = \partial / \partial t (\psi + \phi). \quad (3)$$

PERTURBATION

We are interested in the stability of the electron accumulation layer against perturbations, so we write the electric potential and the quasi Fermi level as the sums

$$\psi = \psi_0(x) + \delta\psi(x,y,z,t)$$

and

$$\phi = \phi_0(x) + \delta\phi(x,y,z,t)$$

of equilibrium parts and perturbations. But the quasi Fermi level Eqn 1 coincides everywhere with the bulk E_F for any equilibrium state regardless of its stability, so $\phi_0 = 0$.

Substituting these expressions into Equations 2 and 3 yields a zeroth order equation for the usual one-dimensional equilibrium

$$\psi_0'' = \exp(\psi_0) - 1$$

and a first order system of equations for the perturbations

$$\nabla^2 \delta\psi = \exp(\psi_0) (\delta\phi + \delta\psi)$$

and

$$\nabla^2 \delta\phi + \psi_0' \delta\phi' = \partial/\partial t (\delta\phi + \delta\psi)$$

where prime (') indicates ordinary differentiation with respect to x.

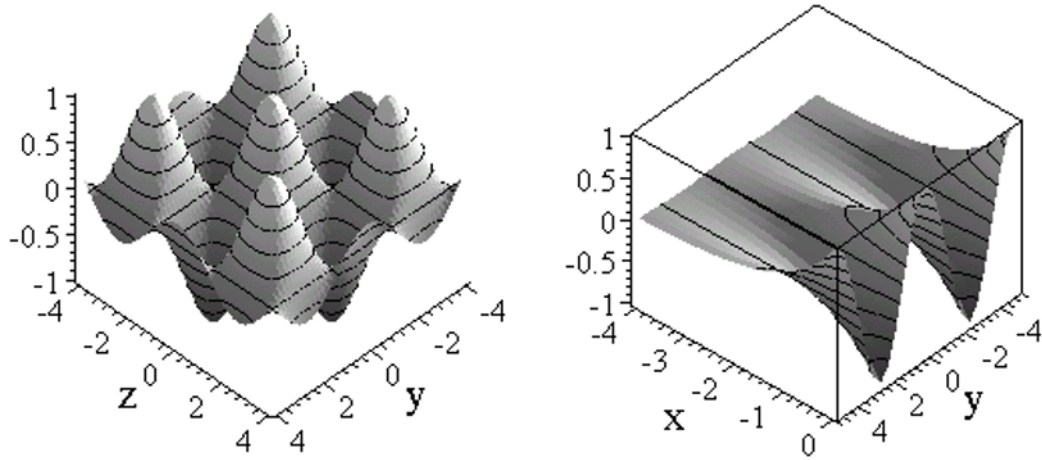


Figure 2. The (real part of the) perturbation $\delta\psi = \psi_1 \exp(ik_x x + ik_y y + Kz + st)$ of the electric potential of the equilibrium accumulation layer at $t=0$ for $\psi_1 = 1$. *Left:* parallel to interface: a slice in the yz plane through $x=0$, and *Right:* normal to interface: a slice through the xy plane through $z=0$.

ANALYTIC APPROXIMATION

The x -dependence of the coefficients $\exp(\psi_0)$ and ψ_0' preclude simple analysis of the first-order system. However the perturbations are confined near the surface where we replace the accumulated electron density with a constant effective density typical of the thin region near the surface

$$\rho \leftarrow \exp(\psi_0)$$

and we replace the slope ψ_0' of the zeroth order potential with a constant effective slope typical of the region

$$\sigma \leftarrow \psi_0'.$$

We estimate the region of validity for this approximation as the x for which $\rho + \sigma x > 0$:

$$-\rho / \sigma < x < 0. \quad (4)$$

In this region of x , the first order equations of motion for perturbations becomes

$$\nabla^2 \delta\psi = \rho (\delta\phi + \delta\psi)$$

and

$$\nabla^2 \delta\phi + \sigma \delta\phi' = \partial/\partial t (\delta\phi + \delta\psi) .$$

We consider perturbations of the form shown in Figure 2

$$\delta\psi = \psi_1 \exp \{ ik_x x + ik_y y + Kz + st \} \quad (5)$$

and

$$\delta\phi = \phi_1 \exp \{ ik_x x + ik_y y + Kz + st \} . \quad (6)$$

where ψ_1 and ϕ_1 are arbitrarily small constants, which are in general complex if there is a phase difference between the two perturbations.

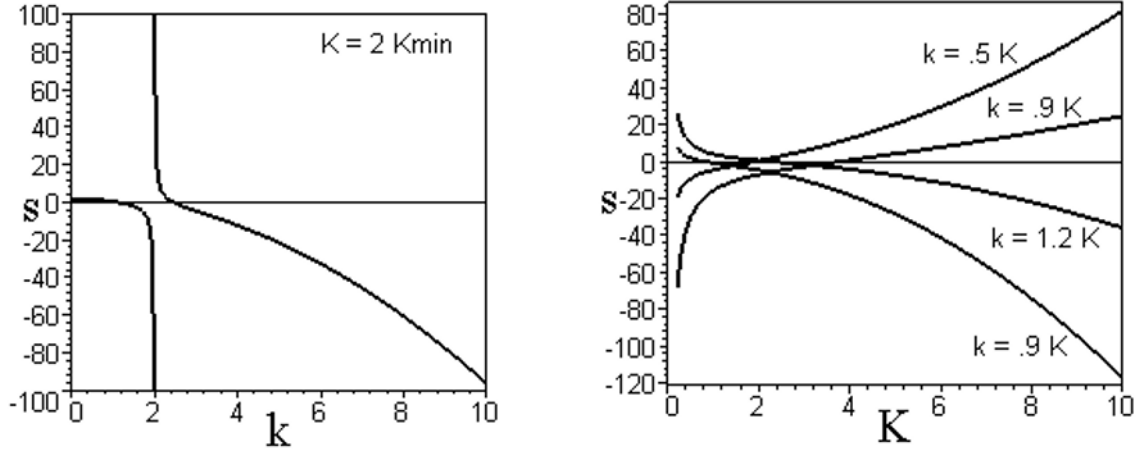


Figure 3. The approximate rate of growth in time of perturbations of wave number k parallel to the cathode surface and spatial decay rate K normal to the cathode surface for the zeroth-order equilibrium $\rho = \exp(1) \approx 2.7$, $\sigma = \rho/2 \approx 1.4$, so that the equilibrium electric potential at the surface is $\psi_0(0) = k_B T \exp(1) \approx 68$ mV relative to the bulk. *Left:* The rate of growth $s(k, K)$ for fixed $K = 1$. *Right:* The rate of growth $s(k, K)$ for four ratios k/K which shows that instabilities may occur in two regimes: large K with low wave number $k < K$ and small K with high wave number $k > K$. All curves satisfy the requirement for validity since $K_{min} = 0.5 < K < 15K_{min} = 7.5$ here.

We can expect the modes modeled by Eqns 5 and 6 to be weakly coupled for K is large enough to confine the perturbations to the region Eqn 4, so that perturbations with

$$K > \sigma / \rho ,$$

satisfy

$$(-k^2 + K^2) \delta\phi = \rho (\delta\phi + \delta\psi)$$

and

$$(-k^2 + K^2 + \sigma K) \delta\psi = s (\delta\phi + \delta\psi)$$

which can be solved for their rate of growth in time

$$s = - (k^2 - K^2 + \rho) (k^2 - K^2 - \sigma K) / (k^2 - K^2) . \quad (7)$$

The one-dimensional equilibrium is stable against any excitation with $s < 0$ since any such perturbation decays exponentially in time back toward the zeroth order equilibrium. On the other hand, modes with positive s grow in time rather than decay so that the one-dimensional zeroth-order equilibrium is unstable against any such perturbation.

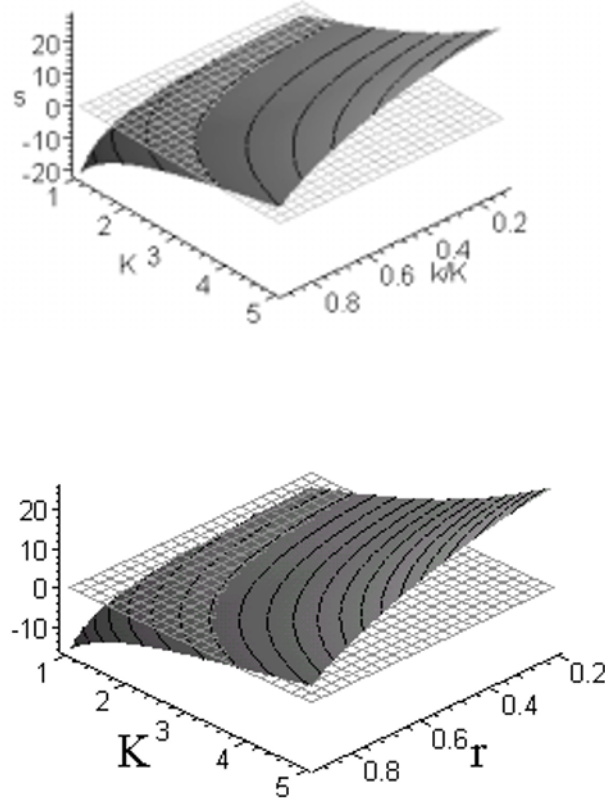


Figure 4. The approximate rate of growth in time of perturbations as a function of spatial decay rate K and the ratio k/K for modes that include the large- K -small- k instabilities shown on the right hand side of Figure 3b for the same $\rho = 2.7$ and $\sigma = 1.4$. The plane $s=0$ has been inserted as a grid.

RESULTS

Eqn 5 predicts positive values of s in some regions of (k, K) space. Figure 3a is a plot of s as a function of K for the case $\rho = 2.7$, $\sigma = 1.4$, and $K=1$. In Figure 3a, perturbations with wave number $k < 1$ or with $k > 1.54$ decay in time but perturbations with $K < k < 1.53$ grow if they are excited. Thermal fluctuations will excite all modes. For $\text{Al}_{0.5}\text{Ga}_{0.5}\text{N}$ doped with silicon at $N_D = 10^{24} \text{ m}^{-3}$, the electron affinity is $\chi = 1 \text{ eV}$, the donor ionization is small, the dielectric permittivity is $\epsilon_S = 8.85 \epsilon_0$, and the electron mobility is $\mu = 30 \text{ V cm}^{-2} \text{ s}^{-1}$ for high quality thin films currently reported, as previously inferred from other literature [4]. In this case $L_D = 3.55 \text{ nm}$ and $\tau_R = 0.16 \text{ ps}$ at room temperature. For Figures 3 and 4 the applied electric field is $F_0 = 1.4 \epsilon_S / \epsilon_0$ in units of VT/L_D and $F_0 = 87 \text{ V}/\mu\text{m}$ in standard units. In this case, we find that the electron accumulation layer may be unstable against variations of a few nanometers wavelength in electron density along the surface.

The regions of high electron density would result in field enhancement and a lowering of the effective barrier for field emission.

REFERENCES

- [1] M.C. Benjamin, M.D. Bremser, T.W. Weeks, Jr., S.W. King, R.F. Davis, and R.J. Nemanich. *Applied Surface Science* **104/105**, 455-460 (1996); Presented at Fifth international Conference on the Formation of Semiconductor Interfaces, Princeton, NJ, June 26-30, 1995.
- [2] J. L. Shaw, H. F. Gray, K. L. Jensen, and T. M. Jung, *J. Vac. Sci. Technol.* **B 14**, 2072 (1996).
- [3] K. L. Jensen, J. E. Yater, E. G. Zaidman, M. A. Kodis, and A. Shih, *J. Vac. Sci. Technol.* **B 16**, 2038 (1998).
- [4] C. W. Hatfield and G. L. Bilbro, *J. Vac. Sci. Technol.* **B 17**, 552 (1999).
- [5] G. L. Bilbro and R. J. Nemanich, *Appl. Phys. Lett.* **Vol. 76** No. 7, 891 (2000).
- [6] A. T. Sowers, B. L. Ward, S. L. English, and R. J. Nemanich, *J. Appl. Phys.* **86**, 3973 (1999).
- [7] R. Stratton, *Proc. Phys. Soc. (London)* **B46**, 746 (1955).